Optimal investment in the development of oil and gas field

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17.05.2021

Scientific supervisor:
Professor A.I. Ersin

Problem statement

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Oil field

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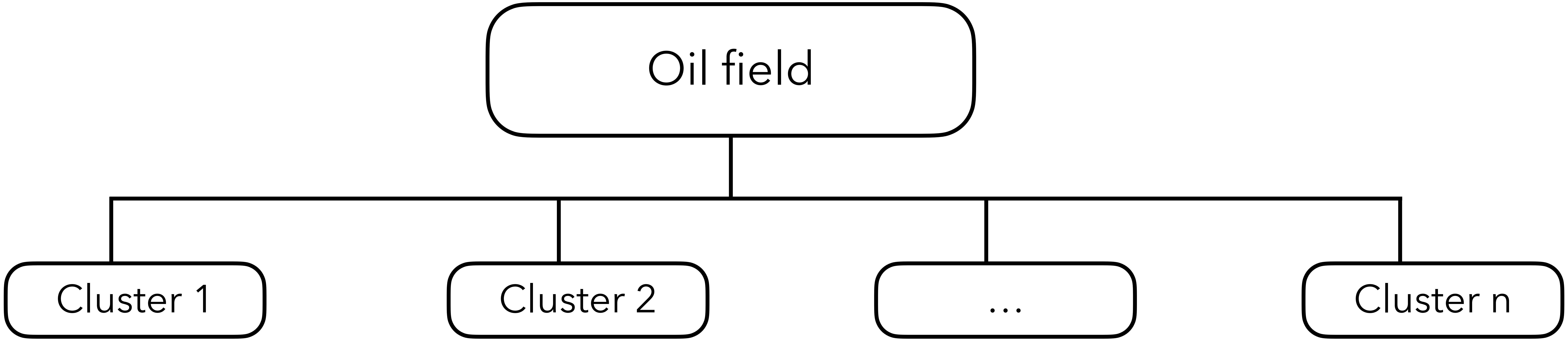

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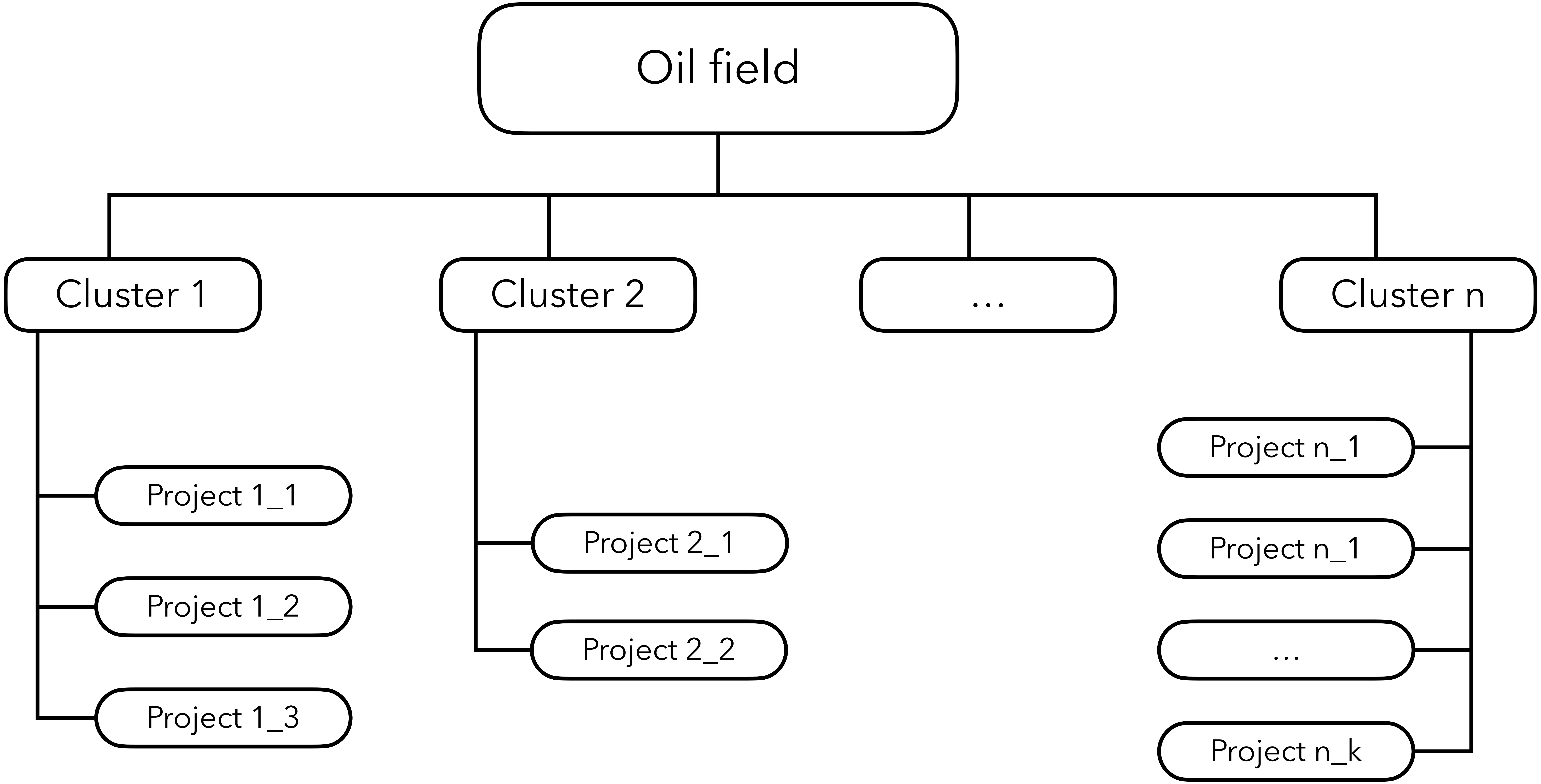

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- $[1, T]$ - the planning period
- C - the total amount of investment
- K - the set of clusters ($|K| = n$)
- P_k - the set of projects for the development of the cluster $k \in K$
- $d_k^i(t)$ - the volume of production in the cluster $k \in K$ per year $t = 1, \ldots, T,$ if the project $i \in P_k$ is implemented here
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And also for variables:

$$x_k^i = \begin{cases} 1, & \text{if project } i \text{ is selected for cluster } k; \\ 0, & \text{else.} \end{cases}$$

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Then the problem under consideration can be written as follows:

$$\sum_{k\in K}\sum_{i\in P_k} q_k^i x_k^i \to \max_{x_k^i\in\{0,1\}}, \tag{1}$$

$$\sum_{k\in K}\sum_{i\in P_k} c_k^i x_k^i \le C, \tag{2}$$

$$\sum_{i\in P_k} x_k^i \le 1,\ k\in K, \tag{3}$$

$$\sum_{k\in K}\sum_{i\in P_k} d_k^i(t) x_k^i \le D(t),\ t\in[1,T]. \tag{4}$$

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Remark 1:

Suppose $d_k^i(t)$ is the volume production per year t if the project i is implemented in the cluster k. If this project is launched τ years later, the annual production during year t will be $d_k^i(t-\tau)$. So, each project in the set P_k is characterized, in particular, by its beginning.

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Remark 2:

Profit from project implementation depends on the year of its launch, as money depreciates over the years. In this regard, at the stage of preliminary calculations, we recount values associated with investment and profit.

Previous research

Markowitz, H.M.: Portfolio Selection. J. of Finance. 7(1), 71-91 (1952)

Markowitz, H.M.: Portfolio Selection: Efficient Diversification of Investment. - Wiley, New York, 1959

Goncharenko, S.N., Safronova, Z.A.: Modeli i metody optimizacii plana dobychi i pervichnoy pererabotki nifti. Gorniy informacionno-analiticheskiy biulleten'. 10, 221-229 (2008) (in Russian)

Huang, S.: An Improved Portfolio Optimization Model for Oil and Gas Investment Selection Considering Investment Period. Open Journal of Social Sciences. 7, 121-129 (2019)

Malah, S.A., Servah, V. V.: O slognosti zadachi vybora investicionih proektov. Vestnik Omskogo universiteta. 3, 10-15 (2016) (in Russian)

The problem without restrictions on production volumes

$$\sum_{k \in K} q_k(c_k) \to \max_{c_k \in [0,C]}, \qquad (5)$$

$$\sum_{k \in K} c_k \le C. \qquad (6)$$

c_k - the amount of the investment allocated for the development of the cluster k

$q_k(c_k)$ - profit function for each cluster k

Solving the problem (5)-(6), we choose «best» project for each cluster.

Approximate algorithm

We will not change the projects selected for each cluster as a result of solving the problem (5)-(6). We will try to determine **the moments of launching these projects** so that inequalities (4) fulfill, and profit takes maximal value.

$$\forall t \in [1,T]: \sum_{k \in K} d_k(t) \leq D(t) .$$

Assume that project whose beginning is shifted by $i \in [0,t_k]$ years in the cluster k - is **another** project.

Then for each cluster, there is a set of projects, which we denote as before by P_k ($|P_k| = t_k + 1$).

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$x_k^i = 1$ if and only if the start of the cluster project k is shifted by i years.

Approximate algorithm

To construct a feasible solution we use **greedy algorithm (GA).**

Suppose we order the projects according to the years of their launch.

π - the permutation of the cluster numbers $\{1,2,\ldots,n\}$

Denote by $P(\pi)$ the list of ordered projects.

1) The first project starts without delay (with zero shift). We exclude it from the $P(\pi)$.
2) For the first project of the updated set $P(\pi)$, we determine its ***earliest**** start time and exclude this project from the set $P(\pi)$.
3) Continue the process until the start year of the last project $\pi(n)$ is found.

* - is no less than the start time of a previous project

Approximate algorithm

Using the **GA**, one can construct a solution for each permutation of the cluster number. Therefore, it is essential to find a permutation where the solution constructed in such way is near-optimal. For this reason, we suggest a **local search** procedure for permutations starting from some *promising* one.

In particular, we can start with $\pi = \{1,2,\ldots,n\}$. At each step, the algorithm iterates through permutations from the neighborhood of the currently selected one: π.

While it is possible to improve the solution, the method iterates over the "neighbors" of the current permutation. We consider two permutations that have the same elements in all positions, except for two, whose values are reversed, to be «neighbors».

Approximate algorithm

Lemma 1

If the order of launching the projects is known, the annual production schedules for all projects are non-increasing, and $D(t) = D = const$, $t \in [1,T]$, then the **GA** determines the optimal start year for all projects.

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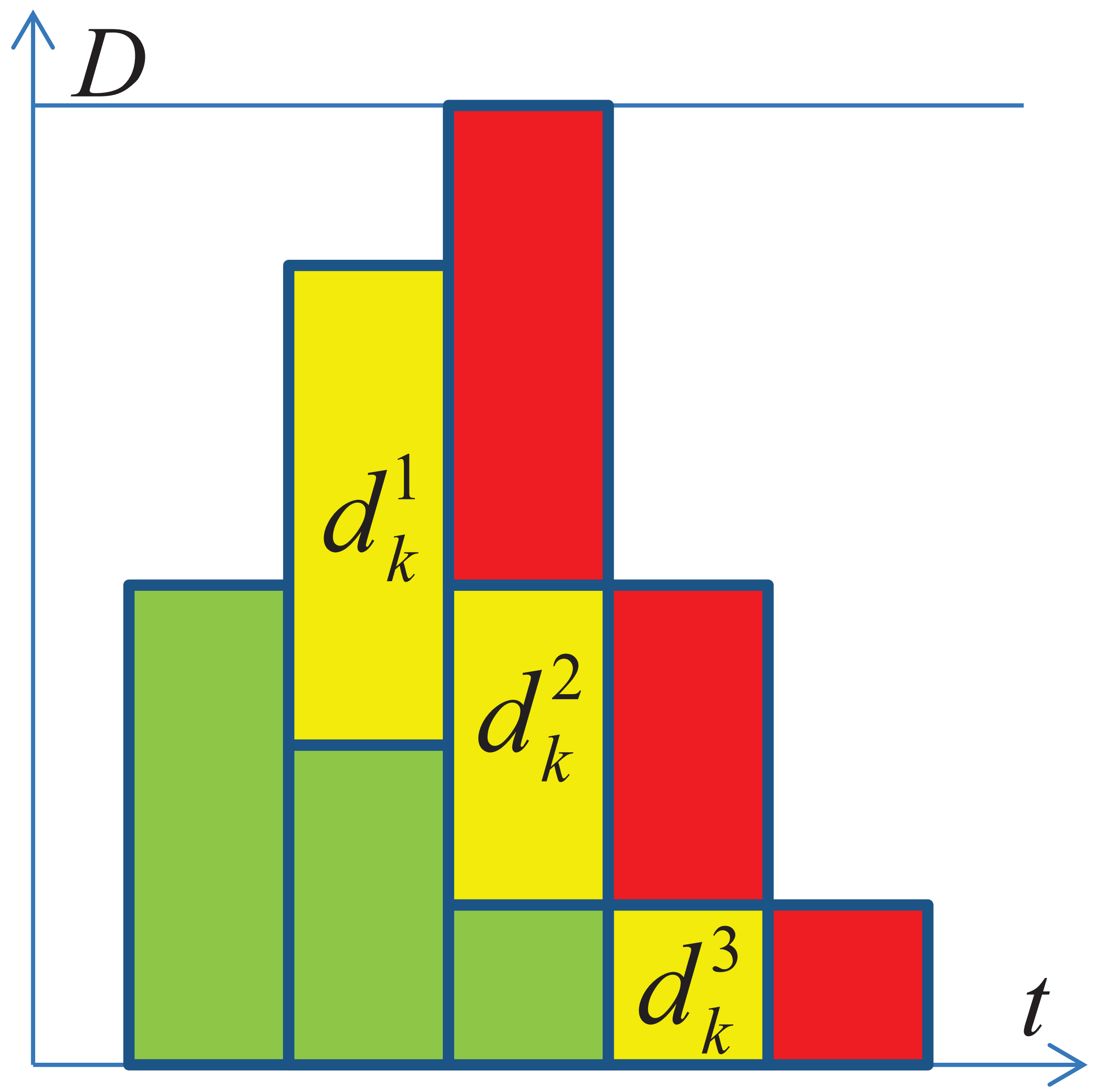

a)

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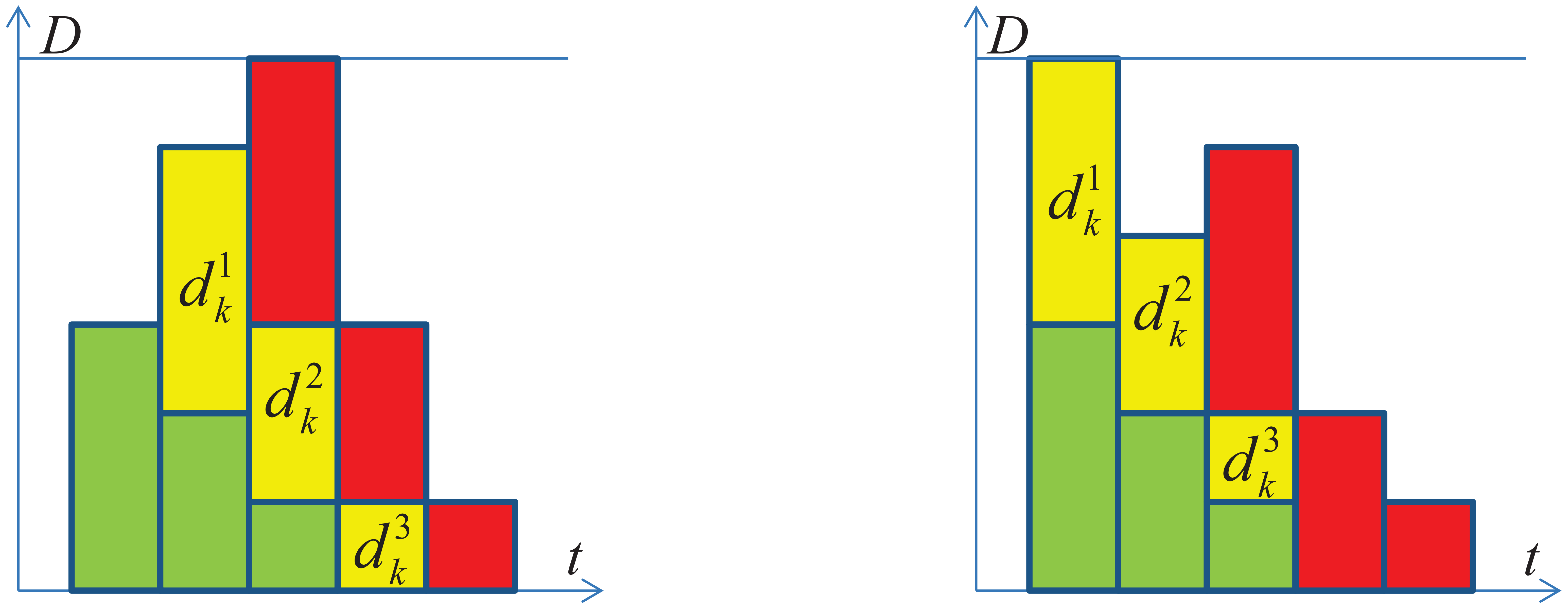

a) *b)*

Simulation

obj - value of the objective function

ub - upper bound

gap - relative difference between *obj* and *ub*

decline - difference between the values of the objective functions *CPLEX* and $CPLEX_{fp}$

$r1 = obj(A)/ub(CPLEX)$

$r2 = obj(A)/ub(CPLEX_{fp})$

estimates from above on the relative error (ratio)

n	$p_{\min}$	$p_{\max}$	$CPLEX$			$CPLEX_{fp}$				A			
			obj	ub	gap	obj	ub	gap	$decline$ (%)	obj	r_1	r_2	time (sec.)
10	1	10	12851.93	12851.93	0	12072	12072	0	6.07	12072	0.94	1	0.006
	10	25	16460.95	16460.95	0	15502.23	15502.23	0	5.82	14710.6	0.89	0.95	0.005
	25	50	16988.53	16988.53	0	14862.28	14862.28	0	12.51	14764.3	0.87	0.99	0.004
	50	100	17140.36	17140.36	0	15607.92	15607.92	0	8.94	15374.1	0.9	0.99	0.003

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25	1	10	30465.26	30465.26	0	29849.9	29849.9	0	2.02	29571.9	0.97	0.99	0.082
	10	25	42501.57	42501.57	0	39728.94	39728.94	0	6.52	38930.6	0.92	0.98	0.077
	25	50	46508.15	46849.94	0.007	43646.48	43646.48	0	6.15	42407.4	0.9	0.97	0.056
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	10	25	86529.19	86659.92	0.002	80616.31	80778.51	0.002	6.83	76845.5	0.89	0.95	0.627
	25	50	93928.28	94290.32	0.003	88415.05	88415.05	0	5.87	86861.8	0.92	0.98	0.612
	50	100	95201.48	95621.93	0.004	88532.86	88661.56	0.001	7	86380.7	0.9	0.97	0.39

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100	1	10	139928.34	140023.11	0.0007	136420.24	136586.12	0.001	2.51	128679	0.92	0.94	5.22
	10	25	173898.61	174065.77	0.001	163833.54	163932.06	0.0006	5.79	154452	0.89	0.94	5.35
	25	50	189722.3	190000.79	0.001	177064.12	177138.3	0.0004	6.67	171431	0.9	0.97	5.84
	50	100	195223.21	195686.39	0.0023	180375.66	180476.58	0.0005	7.61	176830	0.9	0.98	9.44

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	50	100	17140.36	17140.36	0	15607.92	15607.92	0	8.94	15374.1	0.9	0.99	0.003
25	1	10	30465.26	30465.26	0	29849.9	29849.9	0	2.02	29571.9	0.97	0.99	0.082
	10	25	42501.57	42501.57	0	39728.94	39728.94	0	6.52	38930.6	0.92	0.98	0.077
	25	50	46508.15	46849.94	0.007	43646.48	43646.48	0	6.15	42407.4	0.9	0.97	0.056
	50	100	47432.09	47906.72	0.01	44307.47	44307.47	0	6.59	43324	0.9	0.98	0.023
50	1	10	70568.99	70568.99	0	69609.18	69609.18	0	1.36	66328.2	0.94	0.95	0.752
	10	25	86529.19	86659.92	0.002	80616.31	80778.51	0.002	6.83	76845.5	0.89	0.95	0.627
	25	50	93928.28	94290.32	0.003	88415.05	88415.05	0	5.87	86861.8	0.92	0.98	0.612
	50	100	95201.48	95621.93	0.004	88532.86	88661.56	0.001	7	86380.7	0.9	0.97	0.39
100	1	10	139928.34	140023.11	0.0007	136420.24	136586.12	0.001	2.51	128679	0.92	0.94	5.22
	10	25	173898.61	174065.77	0.001	163833.54	163932.06	0.0006	5.79	154452	0.89	0.94	5.35
	25	50	189722.3	190000.79	0.001	177064.12	177138.3	0.0004	6.67	171431	0.9	0.97	5.84
	50	100	195223.21	195686.39	0.0023	180375.66	180476.58	0.0005	7.61	176830	0.9	0.98	9.44

Simulation

n	$p_{\min}$	$p_{\max}$	$CPLEX$			$CPLEX_{fp}$				A			
			obj	ub	gap	obj	ub	gap	$decline$ (%)	obj	r_1	r_2	time (sec.)
10	1	10	12851.93	12851.93	0	12072	12072	0	6.07	12072	0.94	1	0.006
	10	25	16460.95	16460.95	0	15502.23	15502.23	0	5.82	14710.6	0.89	0.95	0.005
	25	50	16988.53	16988.53	0	14862.28	14862.28	0	12.51	14764.3	0.87	0.99	0.004
	50	100	17140.36	17140.36	0	15607.92	15607.92	0	8.94	15374.1	0.9	0.99	0.003
25	1	10	30465.26	30465.26	0	29849.9	29849.9	0	2.02	29571.9	0.97	0.99	0.082
	10	25	42501.57	42501.57	0	39728.94	39728.94	0	6.52	38930.6	0.92	0.98	0.077
	25	50	46508.15	46849.94	0.007	43646.48	43646.48	0	6.15	42407.4	0.9	0.97	0.056
	50	100	47432.09	47906.72	0.01	44307.47	44307.47	0	6.59	43324	0.9	0.98	0.023
50	1	10	70568.99	70568.99	0	69609.18	69609.18	0	1.36	66328.2	0.94	0.95	0.752
	10	25	86529.19	86659.92	0.002	80616.31	80778.51	0.002	6.83	76845.5	0.89	0.95	0.627
	25	50	93928.28	94290.32	0.003	88415.05	88415.05	0	5.87	86861.8	0.92	0.98	0.612
	50	100	95201.48	95621.93	0.004	88532.86	88661.56	0.001	7	86380.7	0.9	0.97	0.39
100	1	10	139928.34	140023.11	0.0007	136420.24	136586.12	0.001	2.51	128679	0.92	0.94	5.22
	10	25	173898.61	174065.77	0.001	163833.54	163932.06	0.0006	5.79	154452	0.89	0.94	5.35
	25	50	189722.3	190000.79	0.001	177064.12	177138.3	0.0004	6.67	171431	0.9	0.97	5.84
	50	100	195223.21	195686.39	0.0023	180375.66	180476.58	0.0005	7.61	176830	0.9	0.98	9.44

Simulation

n	$p_{\min}$	$p_{\max}$	$CPLEX$			$CPLEX_{fp}$				A			
			obj	*ub*	*gap*	*obj*	*ub*	*gap*	*decline* (%)	obj	r_1	r_2	time (sec.)
10	1	10	12851.93	12851.93	0	12072	12072	0	6.07	12072	0.94	1	0.006
	10	25	16460.95	16460.95	0	15502.23	15502.23	0	5.82	14710.6	0.89	0.95	0.005
	25	50	16988.53	16988.53	0	14862.28	14862.28	0	12.51	14764.3	0.87	0.99	0.004
	50	100	17140.36	17140.36	0	15607.92	15607.92	0	8.94	15374.1	0.9	0.99	0.003
25	1	10	30465.26	30465.26	0	29849.9	29849.9	0	2.02	29571.9	0.97	0.99	0.082
	10	25	42501.57	42501.57	0	39728.94	39728.94	0	6.52	38930.6	0.92	0.98	0.077
	25	50	46508.15	46849.94	0.007	43646.48	43646.48	0	6.15	42407.4	0.9	0.97	0.056
	50	100	47432.09	47906.72	0.01	44307.47	44307.47	0	6.59	43324	0.9	0.98	0.023
50	1	10	70568.99	70568.99	0	69609.18	69609.18	0	1.36	66328.2	0.94	0.95	0.752
	10	25	86529.19	86659.92	0.002	80616.31	80778.51	0.002	6.83	76845.5	0.89	0.95	0.627
	25	50	93928.28	94290.32	0.003	88415.05	88415.05	0	5.87	86861.8	0.92	0.98	0.612
	50	100	95201.48	95621.93	0.004	88532.86	88661.56	0.001	7	86380.7	0.9	0.97	0.39
100	1	10	139928.34	140023.11	0.0007	136420.24	136586.12	0.001	2.51	128679	0.92	0.94	5.22
	10	25	173898.61	174065.77	0.001	163833.54	163932.06	0.0006	5.79	154452	0.89	0.94	5.35
	25	50	189722.3	190000.79	0.001	177064.12	177138.3	0.0004	6.67	171431	0.9	0.97	5.84
	50	100	195223.21	195686.39	0.0023	180375.66	180476.58	0.0005	7.61	176830	0.9	0.98	9.44

Simulation

n	$p_{\min}$	$p_{\max}$	$CPLEX$			$CPLEX_{fp}$				A			
			obj	*ub*	*gap*	*obj*	*ub*	*gap*	*decline* (%)	obj	r_1	r_2	time (sec.)
10	1	10	12851.93	12851.93	0	12072	12072	0	6.07	12072	0.94	1	0.006
	10	25	16460.95	16460.95	0	15502.23	15502.23	0	5.82	14710.6	0.89	0.95	0.005
	25	50	16988.53	16988.53	0	14862.28	14862.28	0	12.51	14764.3	0.87	0.99	0.004
	50	100	17140.36	17140.36	0	15607.92	15607.92	0	8.94	15374.1	0.9	0.99	0.003
25	1	10	30465.26	30465.26	0	29849.9	29849.9	0	2.02	29571.9	0.97	0.99	0.082
	10	25	42501.57	42501.57	0	39728.94	39728.94	0	6.52	38930.6	0.92	0.98	0.077
	25	50	46508.15	46849.94	0.007	43646.48	43646.48	0	6.15	42407.4	0.9	0.97	0.056
	50	100	47432.09	47906.72	0.01	44307.47	44307.47	0	6.59	43324	0.9	0.98	0.023
50	1	10	70568.99	70568.99	0	69609.18	69609.18	0	1.36	66328.2	0.94	0.95	0.752
	10	25	86529.19	86659.92	0.002	80616.31	80778.51	0.002	6.83	76845.5	0.89	0.95	0.627
	25	50	93928.28	94290.32	0.003	88415.05	88415.05	0	5.87	86861.8	0.92	0.98	0.612
	50	100	95201.48	95621.93	0.004	88532.86	88661.56	0.001	7	86380.7	0.9	0.97	0.39
100	1	10	139928.34	140023.11	0.0007	136420.24	136586.12	0.001	2.51	128679	0.92	0.94	5.22
	10	25	173898.61	174065.77	0.001	163833.54	163932.06	0.0006	5.79	154452	0.89	0.94	5.35
	25	50	189722.3	190000.79	0.001	177064.12	177138.3	0.0004	6.67	171431	0.9	0.97	5.84
	50	100	195223.21	195686.39	0.0023	180375.66	180476.58	0.0005	7.61	176830	0.9	0.98	9.44

Simulation

n	$p_{\min}$	$p_{\max}$	$CPLEX$			$CPLEX_{fp}$				A			
			obj	*ub*	*gap*	*obj*	*ub*	*gap*	*decline* (%)	obj	r_1	r_2	time (sec.)
10	1	10	12851.93	12851.93	0	12072	12072	0	6.07	12072	0.94	1	0.006
	10	25	16460.95	16460.95	0	15502.23	15502.23	0	5.82	14710.6	0.89	0.95	0.005
	25	50	16988.53	16988.53	0	14862.28	14862.28	0	12.51	14764.3	0.87	0.99	0.004
	50	100	17140.36	17140.36	0	15607.92	15607.92	0	8.94	15374.1	0.9	0.99	0.003
25	1	10	30465.26	30465.26	0	29849.9	29849.9	0	2.02	29571.9	0.97	0.99	0.082
	10	25	42501.57	42501.57	0	39728.94	39728.94	0	6.52	38930.6	0.92	0.98	0.077
	25	50	46508.15	46849.94	0.007	43646.48	43646.48	0	6.15	42407.4	0.9	0.97	0.056
	50	100	47432.09	47906.72	0.01	44307.47	44307.47	0	6.59	43324	0.9	0.98	0.023
50	1	10	70568.99	70568.99	0	69609.18	69609.18	0	1.36	66328.2	0.94	0.95	0.752
	10	25	86529.19	86659.92	0.002	80616.31	80778.51	0.002	6.83	76845.5	0.89	0.95	0.627
	25	50	93928.28	94290.32	0.003	88415.05	88415.05	0	5.87	86861.8	0.92	0.98	0.612
	50	100	95201.48	95621.93	0.004	88532.86	88661.56	0.001	7	86380.7	0.9	0.97	0.39
100	1	10	139928.34	140023.11	0.0007	136420.24	136586.12	0.001	2.51	128679	0.92	0.94	5.22
	10	25	173898.61	174065.77	0.001	163833.54	163932.06	0.0006	5.79	154452	0.89	0.94	5.35
	25	50	189722.3	190000.79	0.001	177064.12	177138.3	0.0004	6.67	171431	0.9	0.97	5.84
	50	100	195223.21	195686.39	0.0023	180375.66	180476.58	0.0005	7.61	176830	0.9	0.98	9.44

Simulation

n	$p_{\min}$	$p_{\max}$	$CPLEX$			$CPLEX_{fp}$				A			
			obj	*ub*	*gap*	*obj*	*ub*	*gap*	*decline* (%)	obj	r_1	r_2	time (sec.)
10	1	10	12851.93	12851.93	0	12072	12072	0	6.07	12072	0.94	1	0.006
	10	25	16460.95	16460.95	0	15502.23	15502.23	0	5.82	14710.6	0.89	0.95	0.005
	25	50	16988.53	16988.53	0	14862.28	14862.28	0	12.51	14764.3	0.87	0.99	0.004
	50	100	17140.36	17140.36	0	15607.92	15607.92	0	8.94	15374.1	0.9	0.99	0.003
25	1	10	30465.26	30465.26	0	29849.9	29849.9	0	2.02	29571.9	0.97	0.99	0.082
	10	25	42501.57	42501.57	0	39728.94	39728.94	0	6.52	38930.6	0.92	0.98	0.077
	25	50	46508.15	46849.94	0.007	43646.48	43646.48	0	6.15	42407.4	0.9	0.97	0.056
	50	100	47432.09	47906.72	0.01	44307.47	44307.47	0	6.59	43324	0.9	0.98	0.023
50	1	10	70568.99	70568.99	0	69609.18	69609.18	0	1.36	66328.2	0.94	0.95	0.752
	10	25	86529.19	86659.92	0.002	80616.31	80778.51	0.002	6.83	76845.5	0.89	0.95	0.627
	25	50	93928.28	94290.32	0.003	88415.05	88415.05	0	5.87	86861.8	0.92	0.98	0.612
	50	100	95201.48	95621.93	0.004	88532.86	88661.56	0.001	7	86380.7	0.9	0.97	0.39
100	1	10	139928.34	140023.11	0.0007	136420.24	136586.12	0.001	2.51	128679	0.92	0.94	5.22
	10	25	173898.61	174065.77	0.001	163833.54	163932.06	0.0006	5.79	154452	0.89	0.94	5.35
	25	50	189722.3	190000.79	0.001	177064.12	177138.3	0.0004	6.67	171431	0.9	0.97	5.84
	50	100	195223.21	195686.39	0.0023	180375.66	180476.58	0.0005	7.61	176830	0.9	0.98	9.44

Simulation

n	$p_{\min}$	$p_{\max}$	$CPLEX$			$CPLEX_{fp}$				A			
			obj	*ub*	*gap*	*obj*	*ub*	*gap*	*decline* (%)	obj	r_1	r_2	time (sec.)
10	1	10	12851.93	12851.93	0	12072	12072	0	6.07	12072	0.94	1	0.006
	10	25	16460.95	16460.95	0	15502.23	15502.23	0	5.82	14710.6	0.89	0.95	0.005
	25	50	16988.53	16988.53	0	14862.28	14862.28	0	12.51	14764.3	0.87	0.99	0.004
	50	100	17140.36	17140.36	0	15607.92	15607.92	0	8.94	15374.1	0.9	0.99	0.003
25	1	10	30465.26	30465.26	0	29849.9	29849.9	0	2.02	29571.9	0.97	0.99	0.082
	10	25	42501.57	42501.57	0	39728.94	39728.94	0	6.52	38930.6	0.92	0.98	0.077
	25	50	46508.15	46849.94	0.007	43646.48	43646.48	0	6.15	42407.4	0.9	0.97	0.056
	50	100	47432.09	47906.72	0.01	44307.47	44307.47	0	6.59	43324	0.9	0.98	0.023
50	1	10	70568.99	70568.99	0	69609.18	69609.18	0	1.36	66328.2	0.94	0.95	0.752
	10	25	86529.19	86659.92	0.002	80616.31	80778.51	0.002	6.83	76845.5	0.89	0.95	0.627
	25	50	93928.28	94290.32	0.003	88415.05	88415.05	0	5.87	86861.8	0.92	0.98	0.612
	50	100	95201.48	95621.93	0.004	88532.86	88661.56	0.001	7	86380.7	0.9	0.97	0.39
100	1	10	139928.34	140023.11	0.0007	136420.24	136586.12	0.001	2.51	128679	0.92	0.94	5.22
	10	25	173898.61	174065.77	0.001	163833.54	163932.06	0.0006	5.79	154452	0.89	0.94	5.35
	25	50	189722.3	190000.79	0.001	177064.12	177138.3	0.0004	6.67	171431	0.9	0.97	5.84
	50	100	195223.21	195686.39	0.0023	180375.66	180476.58	0.0005	7.61	176830	0.9	0.98	9.44

Simulation

n	$p_{\min}$	$p_{\max}$	$CPLEX$ *obj*	*ub*	*gap*	$CPLEX_{fp}$ *obj*	*ub*	*gap*	*decline* (%)	A obj	r_1	r_2	time (sec.)
10	1	10	12851.93	12851.93	0	12072	12072	0	6.07	12072	0.94	1	0.006
	10	25	16460.95	16460.95	0	15502.23	15502.23	0	5.82	14710.6	0.89	0.95	0.005
	25	50	16988.53	16988.53	0	14862.28	14862.28	0	12.51	14764.3	0.87	0.99	0.004
	50	100	17140.36	17140.36	0	15607.92	15607.92	0	8.94	15374.1	0.9	0.99	0.003
25	1	10	30465.26	30465.26	0	29849.9	29849.9	0	2.02	29571.9	0.97	0.99	0.082
	10	25	42501.57	42501.57	0	39728.94	39728.94	0	6.52	38930.6	0.92	0.98	0.077
	25	50	46508.15	46849.94	0.007	43646.48	43646.48	0	6.15	42407.4	0.9	0.97	0.056
	50	100	47432.09	47906.72	0.01	44307.47	44307.47	0	6.59	43324	0.9	0.98	0.023
50	1	10	70568.99	70568.99	0	69609.18	69609.18	0	1.36	66328.2	0.94	0.95	0.752
	10	25	86529.19	86659.92	0.002	80616.31	80778.51	0.002	6.83	76845.5	0.89	0.95	0.627
	25	50	93928.28	94290.32	0.003	88415.05	88415.05	0	5.87	86861.8	0.92	0.98	0.612
	50	100	95201.48	95621.93	0.004	88532.86	88661.56	0.001	7	86380.7	0.9	0.97	0.39
100	1	10	139928.34	140023.11	0.0007	136420.24	136586.12	0.001	2.51	128679	0.92	0.94	5.22
	10	25	173898.61	174065.77	0.001	163833.54	163932.06	0.0006	5.79	154452	0.89	0.94	5.35
	25	50	189722.3	190000.79	0.001	177064.12	177138.3	0.0004	6.67	171431	0.9	0.97	5.84
	50	100	195223.21	195686.39	0.0023	180375.66	180476.58	0.0005	7.61	176830	0.9	0.98	9.44

Simulation

n	$p_{\min}$	$p_{\max}$	$CPLEX$			$CPLEX_{fp}$				A			
			obj	*ub*	*gap*	*obj*	*ub*	*gap*	*decline* (%)	obj	r_1	r_2	time (sec.)
10	1	10	12851.93	12851.93	0	12072	12072	0	6.07	12072	0.94	1	0.006
	10	25	16460.95	16460.95	0	15502.23	15502.23	0	5.82	14710.6	0.89	0.95	0.005
	25	50	16988.53	16988.53	0	14862.28	14862.28	0	12.51	14764.3	0.87	0.99	0.004
	50	100	17140.36	17140.36	0	15607.92	15607.92	0	8.94	15374.1	0.9	0.99	0.003
25	1	10	30465.26	30465.26	0	29849.9	29849.9	0	2.02	29571.9	0.97	0.99	0.082
	10	25	42501.57	42501.57	0	39728.94	39728.94	0	6.52	38930.6	0.92	0.98	0.077
	25	50	46508.15	46849.94	0.007	43646.48	43646.48	0	6.15	42407.4	0.9	0.97	0.056
	50	100	47432.09	47906.72	0.01	44307.47	44307.47	0	6.59	43324	0.9	0.98	0.023
50	1	10	70568.99	70568.99	0	69609.18	69609.18	0	1.36	66328.2	0.94	0.95	0.752
	10	25	86529.19	86659.92	0.002	80616.31	80778.51	0.002	6.83	76845.5	0.89	0.95	0.627
	25	50	93928.28	94290.32	0.003	88415.05	88415.05	0	5.87	86861.8	0.92	0.98	0.612
	50	100	95201.48	95621.93	0.004	88532.86	88661.56	0.001	7	86380.7	0.9	0.97	0.39
100	1	10	139928.34	140023.11	0.0007	136420.24	136586.12	0.001	2.51	128679	0.92	0.94	5.22
	10	25	173898.61	174065.77	0.001	163833.54	163932.06	0.0006	5.79	154452	0.89	0.94	5.35
	25	50	189722.3	190000.79	0.001	177064.12	177138.3	0.0004	6.67	171431	0.9	0.97	5.84
	50	100	195223.21	195686.39	0.0023	180375.66	180476.58	0.0005	7.61	176830	0.9	0.98	9.44
250	250	500	–	–	–	515525.8	516003.7	0.0009	–	495366	–	0.96	366.4

Simulation

n	$p_{\min}$	$p_{\max}$	$CPLEX$			$CPLEX_{fp}$				A			
			obj	*ub*	*gap*	*obj*	*ub*	*gap*	*decline* (%)	obj	r_1	r_2	time (sec.)
10	1	10	12851.93	12851.93	0	12072	12072	0	6.07	12072	0.94	1	0.006
	10	25	16460.95	16460.95	0	15502.23	15502.23	0	5.82	14710.6	0.89	0.95	0.005
	25	50	16988.53	16988.53	0	14862.28	14862.28	0	12.51	14764.3	0.87	0.99	0.004
	50	100	17140.36	17140.36	0	15607.92	15607.92	0	8.94	15374.1	0.9	0.99	0.003
25	1	10	30465.26	30465.26	0	29849.9	29849.9	0	2.02	29571.9	0.97	0.99	0.082
	10	25	42501.57	42501.57	0	39728.94	39728.94	0	6.52	38930.6	0.92	0.98	0.077
	25	50	46508.15	46849.94	0.007	43646.48	43646.48	0	6.15	42407.4	0.9	0.97	0.056
	50	100	47432.09	47906.72	0.01	44307.47	44307.47	0	6.59	43324	0.9	0.98	0.023
50	1	10	70568.99	70568.99	0	69609.18	69609.18	0	1.36	66328.2	0.94	0.95	0.752
	10	25	86529.19	86659.92	0.002	80616.31	80778.51	0.002	6.83	76845.5	0.89	0.95	0.627
	25	50	93928.28	94290.32	0.003	88415.05	88415.05	0	5.87	86861.8	0.92	0.98	0.612
	50	100	95201.48	95621.93	0.004	88532.86	88661.56	0.001	7	86380.7	0.9	0.97	0.39
100	1	10	139928.34	140023.11	0.0007	136420.24	136586.12	0.001	2.51	128679	0.92	0.94	5.22
	10	25	173898.61	174065.77	0.001	163833.54	163932.06	0.0006	5.79	154452	0.89	0.94	5.35
	25	50	189722.3	190000.79	0.001	177064.12	177138.3	0.0004	6.67	171431	0.9	0.97	5.84
	50	100	195223.21	195686.39	0.0023	180375.66	180476.58	0.0005	7.61	176830	0.9	0.98	9.44
250	250	500	–	–	–	515525.8	516003.7	0.0009	–	495366	–	0.96	366.4

Simulation

n	$p_{\min}$	$p_{\max}$	$CPLEX$			$CPLEX_{fp}$				A			
			obj	*ub*	*gap*	*obj*	*ub*	*gap*	*decline* (%)	obj	r_1	r_2	time (sec.)
10	1	10	12851.93	12851.93	0	12072	12072	0	6.07	12072	0.94	1	0.006
	10	25	16460.95	16460.95	0	15502.23	15502.23	0	5.82	14710.6	0.89	0.95	0.005
	25	50	16988.53	16988.53	0	14862.28	14862.28	0	12.51	14764.3	0.87	0.99	0.004
	50	100	17140.36	17140.36	0	15607.92	15607.92	0	8.94	15374.1	0.9	0.99	0.003
25	1	10	30465.26	30465.26	0	29849.9	29849.9	0	2.02	29571.9	0.97	0.99	0.082
	10	25	42501.57	42501.57	0	39728.94	39728.94	0	6.52	38930.6	0.92	0.98	0.077
	25	50	46508.15	46849.94	0.007	43646.48	43646.48	0	6.15	42407.4	0.9	0.97	0.056
	50	100	47432.09	47906.72	0.01	44307.47	44307.47	0	6.59	43324	0.9	0.98	0.023
50	1	10	70568.99	70568.99	0	69609.18	69609.18	0	1.36	66328.2	0.94	0.95	0.752
	10	25	86529.19	86659.92	0.002	80616.31	80778.51	0.002	6.83	76845.5	0.89	0.95	0.627
	25	50	93928.28	94290.32	0.003	88415.05	88415.05	0	5.87	86861.8	0.92	0.98	0.612
	50	100	95201.48	95621.93	0.004	88532.86	88661.56	0.001	7	86380.7	0.9	0.97	0.39
100	1	10	139928.34	140023.11	0.0007	136420.24	136586.12	0.001	2.51	128679	0.92	0.94	5.22
	10	25	173898.61	174065.77	0.001	163833.54	163932.06	0.0006	5.79	154452	0.89	0.94	5.35
	25	50	189722.3	190000.79	0.001	177064.12	177138.3	0.0004	6.67	171431	0.9	0.97	5.84
	50	100	195223.21	195686.39	0.0023	180375.66	180476.58	0.0005	7.61	176830	0.9	0.98	9.44
250	250	500	–	–	–	515525.8	516003.7	0.0009	–	495366	–	0.96	366.4

Simulation

n	$p_{\min}$	$p_{\max}$	$CPLEX$			$CPLEX_{fp}$				A			
			obj	*ub*	*gap*	*obj*	*ub*	*gap*	*decline* (%)	obj	r_1	r_2	time (sec.)
10	1	10	12851.93	12851.93	0	12072	12072	0	6.07	12072	0.94	1	0.006
	10	25	16460.95	16460.95	0	15502.23	15502.23	0	5.82	14710.6	0.89	0.95	0.005
	25	50	16988.53	16988.53	0	14862.28	14862.28	0	12.51	14764.3	0.87	0.99	0.004
	50	100	17140.36	17140.36	0	15607.92	15607.92	0	8.94	15374.1	0.9	0.99	0.003
25	1	10	30465.26	30465.26	0	29849.9	29849.9	0	2.02	29571.9	0.97	0.99	0.082
	10	25	42501.57	42501.57	0	39728.94	39728.94	0	6.52	38930.6	0.92	0.98	0.077
	25	50	46508.15	46849.94	0.007	43646.48	43646.48	0	6.15	42407.4	0.9	0.97	0.056
	50	100	47432.09	47906.72	0.01	44307.47	44307.47	0	6.59	43324	0.9	0.98	0.023
50	1	10	70568.99	70568.99	0	69609.18	69609.18	0	1.36	66328.2	0.94	0.95	0.752
	10	25	86529.19	86659.92	0.002	80616.31	80778.51	0.002	6.83	76845.5	0.89	0.95	0.627
	25	50	93928.28	94290.32	0.003	88415.05	88415.05	0	5.87	86861.8	0.92	0.98	0.612
	50	100	95201.48	95621.93	0.004	88532.86	88661.56	0.001	7	86380.7	0.9	0.97	0.39
100	1	10	139928.34	140023.11	0.0007	136420.24	136586.12	0.001	2.51	128679	0.92	0.94	5.22
	10	25	173898.61	174065.77	0.001	163833.54	163932.06	0.0006	5.79	154452	0.89	0.94	5.35
	25	50	189722.3	190000.79	0.001	177064.12	177138.3	0.0004	6.67	171431	0.9	0.97	5.84
	50	100	195223.21	195686.39	0.0023	180375.66	180476.58	0.0005	7.61	176830	0.9	0.98	9.44
250	250	500	–	–	–	515525.8	516003.7	0.0009	–	495366	–	0.96	366.4

Simulation

n	$p_{\min}$	$p_{\max}$	$CPLEX$			$CPLEX_{fp}$				A			
			obj	*ub*	*gap*	*obj*	*ub*	*gap*	*decline* (%)	obj	r_1	r_2	time (sec.)
10	1	10	12851.93	12851.93	0	12072	12072	0	6.07	12072	0.94	1	0.006
	10	25	16460.95	16460.95	0	15502.23	15502.23	0	5.82	14710.6	0.89	0.95	0.005
	25	50	16988.53	16988.53	0	14862.28	14862.28	0	12.51	14764.3	0.87	0.99	0.004
	50	100	17140.36	17140.36	0	15607.92	15607.92	0	8.94	15374.1	0.9	0.99	0.003
25	1	10	30465.26	30465.26	0	29849.9	29849.9	0	2.02	29571.9	0.97	0.99	0.082
	10	25	42501.57	42501.57	0	39728.94	39728.94	0	6.52	38930.6	0.92	0.98	0.077
	25	50	46508.15	46849.94	0.007	43646.48	43646.48	0	6.15	42407.4	0.9	0.97	0.056
	50	100	47432.09	47906.72	0.01	44307.47	44307.47	0	6.59	43324	0.9	0.98	0.023
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100	1	10	139928.34	140023.11	0.0007	136420.24	136586.12	0.001	2.51	128679	0.92	0.94	5.22
	10	25	173898.61	174065.77	0.001	163833.54	163932.06	0.0006	5.79	154452	0.89	0.94	5.35
	25	50	189722.3	190000.79	0.001	177064.12	177138.3	0.0004	6.67	171431	0.9	0.97	5.84
	50	100	195223.21	195686.39	0.0023	180375.66	180476.58	0.0005	7.61	176830	0.9	0.98	9.44
250	250	500	–	–	–	515525.8	516003.7	0.0009	–	495366	–	0.96	366.4

Simulation

n	$p_{\min}$	$p_{\max}$	$CPLEX$			$CPLEX_{fp}$				A			
			obj	*ub*	*gap*	*obj*	*ub*	*gap*	*decline* (%)	obj	r_1	r_2	time (sec.)
10	1	10	12851.93	12851.93	0	12072	12072	0	6.07	12072	0.94	1	0.006
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100	1	10	139928.34	140023.11	0.0007	136420.24	136586.12	0.001	2.51	128679	0.92	0.94	5.22
	10	25	173898.61	174065.77	0.001	163833.54	163932.06	0.0006	5.79	154452	0.89	0.94	5.35
	25	50	189722.3	190000.79	0.001	177064.12	177138.3	0.0004	6.67	171431	0.9	0.97	5.84
	50	100	195223.21	195686.39	0.0023	180375.66	180476.58	0.0005	7.61	176830	0.9	0.98	9.44
250	250	500	–	–	–	515525.8	516003.7	0.0009	–	495366	–	0.96	366.4

Thank you for your attention!